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A Socratic Dialogue Proving the Square Root of 2 Irrational

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MENO II

A CONTINUATION OF SOCRATES' DIALOGUE WITH MENO
IN WHICH THE BOY PROVES $\sqrt{2}$ IS IRRATIONAL

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Socrates: Well, here we are at the appointed time, Meno.

Meno: Yes, and it looks like a fine day for it, too.

Socrates: And I see our serving boy is also here.

Boy: Yes, I am, and ready to do your bidding.

Socrates: Wonderful. Now, Meno, I want you to be on your guard, as you were the other day, to insure that I teach nothing to the boy, but rather pull out of his mind the premises which are already there.

Meno: I shall do my best, Socrates.

Socrates: I can ask more of no man, Meno, and I am certain that you will do well, and I hope I will give you no call to halt me in my saying if I should say too much, in which you would feel I was actually teaching the boy the answer to this riddle.

Meno: No, Socrates, I don't think I will have to call you on anything you might say today, for the most wondrously learned men of the group of Pythagoras have spent many hours, weeks, and even months and years toiling in their manner to arrive at the mystic solutions to the puzzles formed by the simple squares with which we worked the other day. Therefore, I am certain to regain my virtue, which I lost the other day, when I was so steadfastly proven by you to be in error in my statement that the root of a

square with an area of two square feet was beyond this boy, who is a fine boy, whom we must make to understand that he should do his best here, and not feel that he has done any wrongness by causing me to lose my virtue to you the other day.

Socrates: Meno, my friend, it is my opinion, and I hope it will soon be yours, that your virtue was increased the other day, rather than decreased.

Meno: I fail to see how, when I was humiliated by seeing this young boy, of modest education, arrive in minutes at the highest mystic levels of the magic of the Pythagoreans. Most of all when I wagered as many dinners as you could eat at my house that this could not be the case.

Socrates: First, friend Meno, let me assure you that I will promise never to eat you out of house and home, not that I could if I tried, for my tastes are simple and your wallet is large. Nevertheless, Meno, my friend, I would hasten to add that I will promise, if you like, not to ever come to your table uninvited.

As a second reason you and your virtue should feel better after the events of the other day, because you were in error before, but are less in error now. And the path to virtue, at least one aspect of the path to virtue, is in finding and correcting error.

Meno: Socrates, you know you are always welcome at my table, except when I am suffering from my ulcer, which you aggravate greatly, or at times when I am entertaining the highest nobles of the land, and you would appear out of place in your clothing. (Socrates was known for his simple attire, and for wearing his garments over and over till they wore out. However, the only surviving example of his writing is a laundry list, so we know he kept his clothes clean and somewhat presentable, though simple)

Socrates: I would hope you would have me over because I was a good influence on your development, than for any other reason. I notice you did not respond to my claim to have increased your virtue, through the exorcism of your error.

Meno: Well Socrates, you know that it is not always the easiest thing to give up one's ways, even though one has found them to be in error. Therefore, please forgive me if I am not sounding as grateful as you would like for your lessons.

Socrates: The easier one finds it to give up the ways of error, the easier it is to replace the error with that which we hope is not in error. Is this not the way to virtue?

Meno: Yes, Socrates, and you know the path is hard, and that we often stumble and fall.

Socrates: Yes, but is it not true that we stumble and fall

over the obstacles which we make for ourselves to trip over?

Meno: Certainly that is most true, Socrates, in some cases.

Socrates: Well, then, let us proceed, for I see the hour is upon us when I do my best thinking, and that hour shall be passed soon, and hopefully with it shall pass a bit of your ignorance.

Meno: Well said, Socrates. I am with you.

Socrates: And shall have we a wager on the events of today?

Meno: Certainly, Socrates.

Socrates: And what shall you wager against this boy proving that the length of the root of a square with an area of two square feet, cannot be made by the ratio of two whole numbers?

Meno: You may have anything it is in my power to give, unless it cause harm to myself or to another to give it.

Socrates: Well said, my friend Meno, and I shall leave it at that. And what shall I offer you as a return wager?

Meno: Well, the easiest thing which comes to mind is to wager all those dinners you won from me the other day.

Socrates: Very well, so be it.

Meno: Now Socrates, since you are my friend, I must give you this friendly warning: you know that the Pythagoreans jealously guard their secrets with secret meetings, protected by secret handshakes, secret signs, passwords, and all that, do you not?

Socrates: I have heard as much, friend Meno.

Meno: Then be sure that they will seek revenge upon you for demystifying the ideas and concepts which they worked so long and hard and secretly to create and protect; for they are a jealous lot in the extreme, hiding in mountain caves, which are hardly fit to be called monasteries by even the most hardened monk.

Socrates: I take your meaning, friend Meno, and thank you for your consideration, but I think that if I lose, that they will not bother me, and if I win, it will appear so simple to everyone, that it would be sheerest folly for anyone to make even the smallest gesture to protect its fallen mystic secrecy. Besides, I have a citizen's responsibility to Athens and to all Athenians to do my best to protect them and enlighten them.

Meno: Very well, Socrates. Please do not ever say that I did not try to warn you, especially after they have nailed you to a cross in a public place, where anyone and everyone could hear you say

that the fault of this lay in my name.

Socrates: Do not worry, friend Meno, for if I were not to show this simple feat of logic to you, I should just walk down the street and find someone else, though not someone whose company and conversation I should enjoy as much as yours.

Meno: Thank you, friend Socrates.

Socrates: Now, boy, do you remember me, and the squares with which we worked and played the other day?

Boy: Yes, sir, Socrates.

Socrates: Please, Meno, instruct the boy to merely call me by my name, as does everyone else. Calling me "sir" merely puts me off my mental stride, and, besides, it will create a greater distance between me and the boy.

Meno: You heard what Socrates, said, boy. Can you do it?

Boy: Yes, sir. (Turning to Socrates) You know I like you very much, and that I call you "sir" not only out of relation of our positions in society, but also because of my true respect and admiration, especially after the events of the other day.

Socrates: Yes, boy. And I will try to live up to your expectations. (Turning to Meno) Would you allow some reward for the boy, as well as that which is for myself, if he should prove to your satisfaction that the square root of two is irrational?

Meno: Certainly, Socrates.

Socrates: (taking the boy aside) What would you like the most in the whole world, boy?

Boy: You mean anything?

Socrates: Well, I can't guarantee to get it for you, but at least I can ask it, and it shouldn't hurt to ask; and besides, as you should know, it is very hard to expect someone to give you what you want, if you never let them know you want it.

Boy: Well, Socrates . . . you know what I would want.

Socrates: Do I?

Boy: Better than I knew the square root of two the other day.

Socrates: You want to be a free man, then, and a citizen.

Boy: (looking down) Yes.

Socrates: Don't look down, then, for that is an admirable desire for one to have, and speaks highly of him who has it. I will speak to Meno, while you hold your tongue.

Boy: Yes, Socrates. (bows to kiss his hand, Socrates turns)

Socrates: Friend Meno, how hard do you think it will be for this boy to prove the irrationality of the square root of two?

Meno: You know that I think it is impossible, Socrates.

Socrates: Well, how long did it take the Pythagoreans?

Meno: I should think it took them years.

Socrates: And how many of them were there?

Meno: Quite a few, though not all worked equally, and some hardly at all, for they were most interested in triangles of the right and virtuous variety, and not in squares and their roots.

Socrates: Can you give me an estimate?

Meno: No, I can't say that I can. I am sorry, Socrates.

Socrates: No problem, would you accept five thinkers as an estimate.

Meno: I think that should be fair.

Socrates: And shall we assume they worked for two years, that is the smallest number which retains the plural, and our assumption was that they worked for years.

Meno: Two years is indeed acceptable to me, Socrates.

Socrates: Very well then, Meno, it would appear that the Pythagoreans spent 10 total years of thinking time to solve the riddles of the square root of two.

Meno: I agree.

Socrates: And would you like to hire the Pythagoreans to run your household, Meno?

Meno: Surely I would, Socrates, if they were only for hire, but, as you well know, they are a secret lot, and hire to no one.

Socrates: Well, if I could get you one, perhaps one of the best of them, in fact the leader of the group that solved the square root of two, would you not hire him, and at high wages?

Meno: Certainly, Socrates. I'd be a fool not to.

Socrates: And you would put him in charge of your house.

Meno: And all my lands, too, Socrates.

Socrates: Possibly. Then I would like to propose, that if this boy should solve the proof of the square root of two being irrational, in the next few hours of our discussion, that he be given wages equal to those due to your most highly placed servant for ten years of service, as he shall perform ten years service for you in the next few hours, should he succeed.

Meno: That sounds quite fair, Socrates, I like your logic.

Socrates: (the boy tugs his tunic, to complain that he wants his freedom, not a mere bucket of gold) Hush, boy, did you not promise to hold your tongue?

Boy: Yes, Socrates, but

Socrates: (turning to Meno) And, of course, with the monetary rewards for such a position, go all the rest of it.

Meno: Of course, Socrates. I never thought to cheat you.

Socrates: I know that, friend Meno, but I merely ask for the boy's sake, who is not used to hearing about high finance and the powers and rank which accompany such things.

Meno: Of course, Socrates. Shall I tell the boy what he shall receive?

Socrates: You are very kind to do so, my friend Meno.

Meno: (turns to the boy) You are aware that a servant may not own the amount of gold I would have to give you, should you win the day?

Boy: Yes, sir.

Meno: Therefore, I would have to give to you the freedom to own the money, before I could give you the money, would I not?

Boy: Yes, sir.

Meno: And in giving you freedom, I would be remiss if I did not give you a job and a coming out party of equal position with your wealth, would I not?

Boy: I can't really say, sir, though I suppose so.

Meno: You suppose correctly. I will feed you for a week of partying, and dress you in the finest garments, while you are introduced to the finest ladies and gentlemen of Athens, from

whom you are free to select for your interests as friends, business partners, social acquaintances, connections, and perhaps even a wife, should you find someone you like for that. Do you now understand that there is nothing I would leave out that you would have to ask for, or that if you did have to ask, I would give it immediately, and ask your forgiveness for my error?

Boy: It is hard to understand, but I take your word.

Socrates: Now don't let this all go to your head, boy. This is something you could have figured out for yourself, if you had applied your mind to it as you did to squares the other day. Can you do as well, today?

Boy: I should think and hope so, friend Socrates, for I see you are indeed my friend, and I should hope I am more capable today, for having learned some the other day.

Socrates: We shall see, boy. Let us on to the test. Now you remember the squares we dealt with the other day.

Boy: Yes, Socrates.

Socrates: And the one particular square on the diagonal we made, whose area was two, do you remember that one?

Boy: Yes, Socrates.

Socrates: And you remember that the length of the side of a square, when multiplied by itself, yields the area of the square.

Boy: Everyone at school knows that, Socrates.

Socrates: Well, maybe. However, it is about that side, which when multiplied time itself yields an area of two, that I would like to speak further today. How is that with you?

Boy: That is fine, Socrates. I remember that line, and I sort of liked it the best, if you know what I mean.

Socrates: Good, then we should have a great time. Do you know how long that line is, boy?

Boy: Well, I know that you both thought it wise when I said it was of a length which when made a square of, yielded a square with an area of two, so I suppose I should answer that way.

Socrates: And a good answer it is, too. We are going to make it an even better answer as we proceed.

Boy: Good.

Socrates: Do you remember when you tripped up and fell on

your face the other day, when you thought that the square of area nine was actually a square of area eight?

Boy: Oh yes, Socrates! And I am sorely ashamed, because I still do not know enough to make sure I never make such an error again, and therefore I know my virtue and rightness are lacking.

Socrates: They are not lacking so much that they cannot be improved, are they boy?

Boy: I should hope and pray not.

Socrates: Well today, you are going to tell us some things about that number, which when multiplied by itself gives us two.

Boy: I will tell you everything I know, or think I know, Socrates, and hope that I am correct or can be corrected.

Socrates: To Meno, surely he is a fine boy, eh Meno?

Meno: Yes, I am proud to own him, but I don't see how he can be smart enough to do the work today that would take a Pythagorean monk ten years of cloistered life to accomplish.

Socrates: We shall see. Boy, you are doing fine. I think I could even make a scholar of you, though I fear you might turn to wine and women with your new found wealth, if you succeed, rather than continue to polish the wit which should get you that reward.

Boy: I don't think I would want to spend that much time with women or with wine, Socrates.

Socrates: You will find something, no doubt. So, back to the number which when square gives us two. What can we say about such a number? Is it odd or even? Well it would have to be a whole number to be one of those, would it not, and we saw the other day what happens to whole numbers when they are squared? They give us 1,4,9 and 16 as square areas, did they not?

Boy: Yes, Socrates, though I remember thinking that there should have been a number which would give eight, Socrates?

Socrates: I think we shall find one, if we keep searching. Now, this number, do you remember if it had to be larger or smaller than one?

Boy: Larger, Socrates. For one squared gives only an area of one, and we need an area of two, which is larger.

Socrates: Good. And what of two?

Boy: Two gives a square of four, which is too large.

Socrates: Fine. So the square root of two is smaller than the side two which is the root of four, and larger than the side one which yields one?

Boy: Yes, Socrates.

Socrates: (Turning to Meno) So now he is as far as most of us get in determining the magnitude of the square root of two? And getting farther is largely a matter of guesswork, is it not?

Meno: Yes, Socrates, but I don't see how he will do it.

Socrates: Neither does he. But I do. Watch! (turning to the boy) Now I am going to tell you something you don't know, so Meno will listen very closely to make sure he agrees that I can tell you. You know multiplication, boy?

Boy: I thought I had demonstrated that, Socrates?

Socrates: So you have, my boy, has he not Meno?

Meno: Yes, Socrates, I recall he did the other day.

Socrates: And you know the way to undo multiplication?

Boy: It is called division, but I do not know it as well as multiplication, since we have not studied it as long.

Socrates: Well, I will not ask you to do much division, but rather I will ask you only whether certain answers may be called odd or even, and the like. Does that suit you?

Boy: It suits me well, Socrates.

Socrates: Then you know what odd and even are, boy?

Boy: Yes, shall I tell you?

Socrates: Please do. I would love to hear what they teach.

Boy: (the boy recites) A number can only be odd or even if it is a whole number, that is has no parts but only wholes of what it measures. Even numbers are special in that they have only whole twos in them, with no ones left over, while odd numbers always have a one left over when all the twos are taken out.

Socrates: An interesting, and somewhat effective definition. Do you agree, Meno.

Meno: Yes, Socrates. Please continue.

Socrates: Now boy, what do you get when you divide these odd and even numbers by other odd and even numbers.

Boy: Sometimes you get whole numbers, especially when you divide an even number by an even number, but odd numbers sometimes give whole numbers, both odd and even, and sometimes they give numbers which are not whole numbers, but have parts.

Socrates: Very good, and have your teachers ever called these numbers ratios?

Boy: Sometimes, Socrates, but usually only with simple numbers which make one-half, one-third, two-thirds and the like.

Socrates: Yes, that is usually what people mean by ratios. The learned people call numbers made from the ratios, rational. Does the name rational number suit you to call a number which can be expressed as the ratio of two whole numbers, whether they be odd or even whole numbers?

Boy: You want me to call the numbers made from ratios of whole numbers something called rational? A ratio makes a rational number?

Socrates: Yes boy, can you do that?

Boy: Certainly, Socrates.

Socrates: Do you agree with the way I told him this, Meno? Does it violate our agreement?

Meno: You added -nal to the word ratio, just as we add -nal to the French word "jour" to create the word journal which means something that contains words of the "jour" or of today. So we now have a word which means a number made from a ratio. This is more than acceptable to me, Socrates. A sort of lesson in linguistics, perhaps, but certainly not in mathematics. No, I do not see that you have told him how to solve anything about the square root of two, but thank you for asking. I give you your journalistic license to do so.

Socrates: Good. Now boy, I need your attention. Please get up and stretch, if it will help you stay and think for awhile.

Boy: (stretches only a little) I am fine, Socrates.

Socrates: Now think carefully, boy, what kind of ratios can we make from even numbers and odd numbers?

Boy: We could make even numbers divided by odd numbers, and odd numbers divided by even numbers.

Socrates: Yes, we could. Could we make any other kind?

Boy: Well . . . we could make even numbers divided by even

numbers, or odd numbers divided by odd.

Socrates: Very good. Any other kind?

Boy: I'm not sure, I can't think of any, but I might have to think a while to be sure.

Socrates: (to Meno) Are you still satisfied.

Meno: Yes, Socrates. He knows even and odd numbers, and ratios; as do all the school children his age.

Socrates: Very well, boy. You have named four kinds of ratios: Even over odd, odd over even, even over even, odd over odd, and all the ratios make numbers we call rational numbers.

Boy: That's what it looks like, Socrates.

Socrates: Meno, have you anything to contribute here?

Meno: No, Socrates, I am fine.

Socrates: Very well. Now, boy, we are off in search of more about the square root of two. We have divided the rational numbers into four groups, odd/even, even/odd, even/even, odd/odd?

Boy: Yes.

Socrates: And if we find another group we can include them. Now, we want to find which one of these groups, if any, contains the number you found the other day, the one which squared is two.

Would that be fun to try?

Boy: Yes, Socrates, and also educational.

Socrates: I think we can narrow these four groups down to three, and thus make the search easier. Would you like that?

Boy: Certainly, Socrates.

Socrates: Let's take even over even ratios. What are they?

Boy: We know that both parts of the ratio have two in them.

Socrates: Excellent. See, Meno, how well he has learned his lessons in school. His teacher must be proud, for I have taught him nothing of this, have I?

Meno: No, I have not seen you teach it to him, therefore he must have been exposed to it elsewhere.

Socrates: (back to the boy) And what have you learned about

ratios of even numbers, boy?

Boy: That both parts can be divided by two, to get the twos out, over and over, until one part becomes odd.

Socrates: Very good. Do all school children know that, Meno?

Meno: All the ones who stay awake in class. (he stretches)

Socrates: So, boy, we can change the parts of the ratios, without changing the real meaning of the ratio itself?

Boy: Yes, Socrates. I will demonstrate, as we do in class. Suppose I use 16 and 8, as we did the other day. If I make a ratio of 16 divided by 8, I can divide both the 16 and the 8 by two and get 8 divided by 4. We can see that 8 divided by 4 is the same as 16 divided by 8, each one is twice the other, as it should be. We can then divide by two again and get 4 over 2, and again to get 2 over 1. We can't do it again, so we say that this fraction has been reduced as far as it will go, and everything that is true of the other ways of expressing it is true of this.

Socrates: Your demonstration is effective. Can you divide by other numbers than two?

Boy: Yes, Socrates. We can divide by any number which goes as wholes into the parts which make up the ratio. We could have started by dividing by 8 before, but I divided by three times, each time by two, to show you the process, though now I feel ashamed because I realize you are both masters of this, and that I spoke to you in too simple a manner.

Socrates: Better to speak too simply, than in a manner in which part or all of your audience gets lost, like the Sophists.

Boy: I agree, but please stop me if I get too simple.

Socrates: I am sure we can survive a simple explanation. (nudges Meno, who has been gazing elsewhere) But back to your simple proof: we know that a ratio of two even numbers can be divided until reduced until one or both its parts are odd?

Boy: Yes, Socrates. Then it is a proper ratio.

Socrates: So we can eliminate one of our four groups, the one where even was divided by even, and now we have odd/odd, odd/even and even/odd?

Boy: Yes, Socrates.

Socrates: Let's try odd over even next, shall we?

Boy: Fine.

Socrates: What happens when you multiply an even number by an even number, what kind of number do you get, even or odd?

Boy: Even, of course. An even multiple of any whole number gives another even number.

Socrates: Wonderful, you have answered two questions, but we need only one at the moment. We shall save the other. So, with odd over even, if we multiply any of these times themselves, we will get odd times odd over even times even, and therefore odd over even, since odd times odd is odd and even of even is even.

Boy: Yes. A ratio of odd over even, when multiplied times itself, yields odd over even.

Socrates: And can our square root of two be in that group?

Boy: I don't know, Socrates. Have I failed?

Socrates: Oh, you know, you just don't know that you know.

Try this: after we multiply our number times itself, which the learned call "squaring" the number which is the root, we need to get a ratio in which the first or top number is twice as large as the second or bottom number. Is this much correct?

Boy: A ratio which when "squared" as you called it, yields an area of two, must then yield one part which is two times the other part. That is the definition of a ratio of two to one.

Socrates: So you agree that this is correct?

Boy: Certainly.

Socrates: Now if a number is to be twice as great as another, it must be two times that number?

Boy: Certainly.

Socrates: And if a number is two times any whole number, it must then be an even number, must it not?

Boy: Yes, Socrates.

Socrates: So, in our ratio we want to square to get two, the top number cannot be odd, can it?

Boy: No, Socrates. Therefore, the group of odd over even rational numbers cannot have the square root of two in it! Nor can the group ratios of odd numbers over odd numbers.

Socrates: Wonderful. We have just eliminated three of the

four groups of rational numbers, first we eliminated the group of even over even numbers, then the ones with odd numbers divided by other numbers. However, these were the easier part, and we are now most of the way up the mountain, so we must rest and prepare to try even harder to conquer the rest, where the altitude is highest, and the terrain is rockiest. So let us sit and rest a minute, and look over what we have done, if you will.

Boy: Certainly, Socrates, though I am much invigorated by the solution of two parts of the puzzle with one thought. It was truly wonderful to see such simple effectiveness. Are all great thoughts as simple as these, once you see them clearly?

Socrates: What do you say, Meno? Do thoughts get simpler as they get greater?

Meno: Well, it would appear that they do, for as the master of a great house, I can just order something be done, and it is; but if I were a master in a lesser house, I would have to watch over it much more closely to insure it got done. The bigger the decisions I have to make, the more help and advice I get in the making of them, so I would have to agree.

Socrates: Glad to see that you are still agreeable, Meno, though I think there are some slight differences in the way each of us view the simplicity of great thought. Shall we go on?

Meno: Yes, quite.

Boy: Yes, Socrates. I am ready for the last group, the ratios of even numbers divided by the odd, though, I cannot yet see how we will figure these out, yet, somehow I have confidence that the walls of these numbers shall tumble before us, as did the three groups before them.

Socrates: Let us review the three earlier groups, to prepare us for the fourth, and to make sure that we have not already broken the rules and therefore forfeited our wager. The four groups were even over even ratios, which we decided could be reduced in various manners to the other groups by dividing until one number of the ratio was no longer even; then we eliminated the two other groups which had odd numbers divided by either odd or even numbers, because the first or top number had to be twice the second or bottom number, and therefore could not be odd; this left the last group we are now to greet, even divided by odd.

Boy: Wonderfully put, Socrates. It is amazing how neatly you put an hour of thinking into a minute. Perhaps we can, indeed, put ten years of thinking into this one day. Please continue in this manner, if you know how it can be done.

Socrates: Would you have me continue, Meno? You know what shall have to happen if we solve this next group and do not

find the square root of two in it.

Meno: Socrates, you are my friend, and my teacher, and a good companion. I will not shirk my duty to you or to this fine boy, who appears to be growing beyond my head, even as we speak. However, I still do not see that his head has reached the clouds wherein lie the minds of the Pythagoreans.

Socrates: Very well, on then, to even over odd. If we multiply these numbers times themselves, what do we get, boy?

Boy: We will get a ratio of even over odd, Socrates.

Socrates: And could an even number be double an odd number?

Boy: Yes, Socrates.

Socrates: So, indeed, this could be where we find a number such that when multiplied times itself yields an area of two?

Boy: Yes, Socrates. It could very well be in this group.

Socrates: So, the first, or top number, is the result of an even number times itself?

Boy: Yes.

Socrates: And the second, or bottom number, is the result of an odd number times itself?

Boy: Yes.

Socrates: And an even number is two times one whole number?

Boy: Of course.

Socrates: So if we use this even number twice in multiplication, as we have on top, we have two twos times two whole numbers?

Boy: Yes, Socrates.

Socrates: (nudges Meno) and therefore the top number is four times some whole number times that whole number again?

Boy: Yes, Socrates.

Socrates: And this number on top has to be twice the number on the bottom, if the even over odd number we began with is to give us two when multiplied by itself, or squared, as we call it?

Boy: Yes, Socrates.

Socrates: And if the top number is four times some whole number,

then a number half as large would have to be two times that same whole number?

Boy: Of course, Socrates.

Socrates: So the number on the bottom is two times that whole number, whatever it is?

Boy: Yes, Socrates.

Socrates: (standing) And if it is two times a whole number, then it must be an even number, must it not?

Boy: Yes.

Socrates: Then it cannot be a member of the group which has an odd number on the bottom, can it?

Boy: No, Socrates.

Socrates: So can it be a member of the ratios created by an even number divided by an odd number and then used as a root to create a square?

Boy: No, Socrates. And that must mean it can't be a member of the last group, doesn't it?

Socrates: Yes, my boy, although I don't see how we can continue calling you boy, since you have now won your freedom, and are far richer than I will ever be.

Boy: Are you sure we have proved this properly? Let me go over it again, so I can see it in my head.

Socrates: Yes, my boy, er, ah, sir.

Boy: We want to see if this square root of two we discovered the other day is a member of the rational numbers?

Socrates: Yes.

Boy: So we define the rational numbers as numbers made from the division into ratios of whole numbers, whether those whole numbers are even or odd.

Socrates: Yes.

Boy: We get four groups, even over even, which we don't use, odd over even, odd over odd, and even over odd.

Socrates: Continue.

Boy: We know the first number in the squared ratio cannot be odd because it must be twice the value of the second number,

and therefore it must be an even number, two times a whole number. Therefore it cannot be a member of either of the next groups, because they both have whole numbers over odd numbers.

Socrates: Wonderful!

Boy: So we are left with one group, the evens over odds.

Socrates: Yes.

Boy: When we square an even over odd ratio, the first number becomes even times even, which is two times two times some other whole number, which means it is four times the whole number, and this number must be double the second number, which is odd, as it was made of odd times odd. But the top number cannot be double some bottom odd number because the top number is four times some whole number, and the bottom number is odd--but a number which is four times another whole number, cannot be odd when cut in half, so an even number times an even number can never be double what you would get from any odd number times another odd number. . .

therefore none of these rational numbers, when multiplied times themselves, could possibly yield a ratio in which the top number was twice the bottom number. Amazing. We have proved that the square root of two is not a rational number. Fantastic!

(he continues to wander up and down the stage, reciting various portions of the proof to himself, looking up, then down, then all around. He comes to Meno)

Boy: Do you see? It's so simple, so clear. This is really wonderful! This is fantastic!

Socrates: (lays an arm on Meno's arm) Tell him how happy you are for his new found thoughts, Meno, for you can easily tell he is not thinking at all of his newly won freedom and wealth.

Meno: I quite agree with you, son, the clarity of your reasoning is truly astounding. I will leave you here with Socrates, as I go to prepare my household. I trust you will both be happy for the rest of the day without my assistance.

[The party, the presentation of 10 years salary to the newly freed young man, is another story, as is the original story of the drawing in the sand the square with an area of two.]

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